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SOME CERTAIN SUMMATION FORMULA OF Q- SERIES AND Q–CONTINUED FRACTIONS

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Abstract: In this paper author try to establish transformation formulas using hyper geometric functions. In order to derive these transformations, two well-known methods are used i.e., the q-series and q-continued factions. Main objective is to establish transformation formulas using hyper geometric functions with the help of known transformations formulas in hyper geometric functions. Some more transformation formulas can be established using known hyper geometric functions, continued fractions of two hyper geometric functions can be established with the help of q- fractional operators. The focus of this paper is on hyper geometric functions, which are special functions and solution of a specific second order linear differential equation. We express these hyper geometric functions in terms of their integral representations. Over the years researchers have worked upon some approximations of these interesting integrals, given one of the parameters in the hyper geometric function is large. However, there was no unified analysis of all the cases of these integrals.

Keywords: q-series; continued fraction identities.

Introduction

From the early age of mathematics Continued fractions have been playing a very important role in number theory and classical analysis from the time of Euler and Gauss. Generalized hypergeometric series, both q-series and q-fractions,have been a very significant tool in the derivation of continued fraction representations. Cao and Srivastava (2013), ChoiandSrivastava (2014), Luo and Srivastava (2011), Srivastava (2011), Srivasatava and Choi (2012) and Srivastava and Choudhary (2015) for useful and interesting similar results.In what follows, we shalluse the following usual notations and definitions. Some interesting applications in the direction of quantum calculus can be seen in Mishra, Khatri, Mishra, and Deepmala (2013), Mishra, Khan, Khatri,and Mishra (2013), Mishra, Khatri, and Mishra (2012, 2013a, 2013b), Mishra, Sharma, and Mishra(2016), Gairola, Deepmala, and Mishra (2016a, 2016b), and Singh, Gairola, and Deepmala (2016).

Extensions of Gamma, Beta, Gauss Hyper geometric function (GHF) and confluent hyper geometric function (CHF) have been extensively studied in the recent past by inserting a regularization factor $e^{-p/t}$.

The following extension of the gamma function is introduced by Chaudhry and Zubair

$$\Gamma_p(x) = \int_0^\infty t^{x-1} exp\left(-t - \frac{p}{t}\right) dt, Re(p) > 0.$$

The extension of Euler's beta function is considered by Chaudhry et al. in thefollowing form

$$\beta_p(x,y) = \int_0^1 t^{x-1}(1-t)^{y-1} exp\left(\frac{-p}{t(1-t)}\right) dt,$$
$$Re(p) > 0, Re(x) > 0, Re(y) > 0,$$

and they proved that this extension has connections with the Macdonald, error andWhittaker functions; and as a result

$$\Gamma_0(x) = \Gamma(x) \; and \; \beta_p(x,y) = \beta(x,y).$$

Following this, Chaudhry et al. used βp(x,y) , to extend the hypergeometric function, known as the extended Gauss hypergeometric function (EGHF), as follows

$$F_p(a,b;c;z) = \sum_{n=0}^{\infty} (a)_n \frac{\beta_p(b+n, c-b)(z)^n}{\beta(b, c-b)\, n!},$$
$$p \geq 0, Re(c) > Re(b) > 0,$$

where (a)n denotes the Pochhammer symbol defined by

$$(a)_n = \frac{\Gamma(a+n)}{\Gamma(a)} = \begin{cases} 1, n = 0; a \in \mathbb{C} \,/\, \{0\} \\ a(a+1)(a+2)\ldots(a+n-1), \qquad n \in \mathbb{N}, a \in \mathbb{C}. \end{cases}$$

Result and Discussion:

Hyper geometric Series by Using Bailey's Transform

The Bailey's transform is obtained from a suitably modified G terminating very well poised summation theorem and term wise transformation. It is been interpreted as a matrix inversion result of two infinite, lower triangular matrixes. This provides a higher dimensional generalization of Andrew's matrix inversion formulation of Bailey's transformation.

W. N. Bailey, in 1947 introduced the following transformation formula. If

$$\beta_n = \sum_{r=0}^{n} \alpha_r u_{n-r} v_{n+r} \,,$$

And

$$\gamma_n = \sum_{r=n}^{\infty} \delta_r u_{r-n} v_{r+n} \,,$$

Then

$$\sum_{n=0}^{\infty} \alpha_n \gamma_n = \sum_{n=0}^{\infty} \beta_p \delta_n \,.$$

where αr , δr , ur and vr are functions of r only, such that the series for n exists. This transformation leads to various results which play an important role in number theory and transformation theory of ordinary and basic hypergeometric series both. Making use of the transformation, Bailey [] developed technique to obtain transformations for both ordinary and basic hypergeometric series and successfully used these transformations to obtain a number of identities of Rogers-Ramanujan type. In 1951 and 1952, L. J. Slater derived one hundred and thirty identities of Rogers-Ramanujan type with the help of Bailey transformation formula.

Extended Bailey's transform:

$$\beta_n = \sum_{r=0}^{[n/p]} \alpha_r u_{n-pr} v_{n+r} t_{n-r} w_{n+pr} ,$$

$$\gamma_n = \sum_{r=pn}^{\infty} \delta_r u_{r-pn} v_{r+n} t_{r-n} w_{r+pn} ,$$

then, subject to convergence conditions,

$$\sum_{n=0}^{\infty} \alpha_n \gamma_n = \sum_{n=0}^{\infty} \beta_n \delta_n .$$

where p is any integer and αr , δr , ur , vr , tr, and wr are any functions of r only. Obviously, the p = 1 case is the original Bailey's transform.

Finally Using Jordan's identity for the bilateral series:

For the bilateral series (Jordan-Kronecker Function), defined as

$$f(x,t) = \sum_{n=-\infty}^{+\infty} \frac{t^n}{1-xq^n}$$

we have f(x, t) = f(t, x) and the following two relations given as

$$f(x,t) = \sum_{n=-\infty}^{+\infty} q^{n^2} x^n t^n \left(+1 + \frac{xq^n}{1-xq^n} + \frac{tq^n}{1-tq^n}\right)$$

$$f(x,t) = \sum_{n=-\infty}^{+\infty} q^{n^2} x^n t^n \left(-1 + \frac{1}{1-xq^n} + \frac{1}{1-tq^n}\right)$$

Above both forms are equivalent to

$$f(x,t) = \sum_{n=-\infty}^{+\infty} \frac{(1-txq^{2n})}{(1-xq^n)(1-tq^n)} x^n t^n q^{n^2}$$

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